H 10.02 – Compounding Name: _____

Use this assignment to help your understanding of compounding a loan.

Situation: You borrow $40,000 from a bank to buy a car. Cool. You will pay back the loan after six years. The interest rate is 5%.

The amount that you will actually pay back is based on the number of times that the bank calculates, or applies, the interest to your debt. This is called ***compounding***. In the beginning, your debt is $40,000.

*Case 1 – Interest calculated **once**, at the end of the term of the loan.*

How it works: Existing Value of the Debt × (1 + Interest rate) = New Value of the Debt

$40,000 x (1 + 0.05) = $42,000.

This means you only have to pay back the original you borrowed plus 5% overall. You borrowed $40,000, you pay back $42,000. This never happens from a bank.

Task: Fill in the tables and answer the questions below:

*Case 2 – Interest calculated **annually** (at the end of each year)*

To Do

a) Multiply straight across: Existing Value × Interest Applied = New Value

b) Then carry the New Value down to the Existing Value for the next year (red arrows). The value of the debt has grown. Then you apply the interest rate for that year to that New Existing Value amount.

Year	Existing Value of the Debt	Interest Applied × (1 + 0.05)	New Value of the Debt	Change
0			$ 40,000	N / A
1	$ 40,000	× *1.05*	$ 42,000	$2,000
2	$ 42,000	× *1.05*	$ 44,100	$2,100
3		× *1.05*		
4		× *1.05*		
5		× *1.05*		
6		× *1.05*		

Question:

a) What is happening to the Change in value of the Debt? Why is that happening?

b) How much more do you have to pay back this way compared to if the interest was only applied once at the end of the six-year term?

*Case 3 – Interest calculated **quarterly** (four times a year)*

First, read this table. Down the left-hand side you see the time period. Quarterly means four times per year. This means the interest is applied at the end of March, then at the end of June, then at the end of September, then again in December. Q 1 means the 1st Quarter, etc. Note that the ***quarterly interest rate*** is not the same as the annual interest rate; it's the annual rate divided by 4. Mechanics of the table are the same.

Quarter	Existing Value of the Debt	Interest Applied $\times \left(1 + \frac{0.05}{4}\right)$	New Value of the Debt	Change
0			$ 40,000	N / A
Year 1, Q 1 (March)	$ 40,000	× *1.0125*	$ 40,500	$ 500
Year 1, Q 2 (June)	$ 40,500	× *1.0125*	$ 41,006	$ 506
Year 1, Q 3 (Sept.)		× *1.0125*		
Year 1, Q 4 (Dec.)		× *1.0125*		
Year 2, Q 1 (March)		× *1.0125*		
Year 2, Q 2 (June)		× *1.0125*		
Year 2, Q 3 (Sept.)		× *1.0125*		
Year 2, Q 4 (Dec.)		× *1.0125*		

Questions:

a) Compare the amount of the debt at the end of Year 1 based on this table compared to the Annual Table on page 1. Which amount is higher? By how much?

b) Compare the amount of the debt at the end of Year 2 based on this table compared to the Annual Table on page 1. Which amount is higher? By how much?

c) Why?

It's annoying to make tables like this, so we use exponential functions instead. Below is the string that would give us the result at the end of Year 1.

$40,000 x (1.0125) x (1.0125) x (1.0125) x (1.0125) = $42,037.81 or $40,000 x $(1.0125)^4$ = $42,037.81

This is about as "real world" as your math classes are ever going to get.